



# Approximate Homomorphic Encryption

- Construction & Bootstrapping

Yongsoo Song, Seoul National Univ

ECC 2018, Osaka



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Yongsoo Song, ~~Seoul National Univ~~  
~~UC San Diego~~  
Microsoft Research, Redmond

**ECC 2018, Osaka**

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- **Construction**
  - [CKKS, AC17] Homomorphic Encryption for Arithmetic of Approximate Numbers
- **Bootstrapping**
  - [CHKKS, EC18] Bootstrapping for Approximate Homomorphic Encryption
- **Related Works**

# Advanced Cryptography

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- Protecting Computation, not just data



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- Protecting Computation, not just data
- Differential Privacy
- Zero-knowledge Proof
- Multiparty Computation
- Attribute Based Encryption
- ...



# Advanced Cryptography

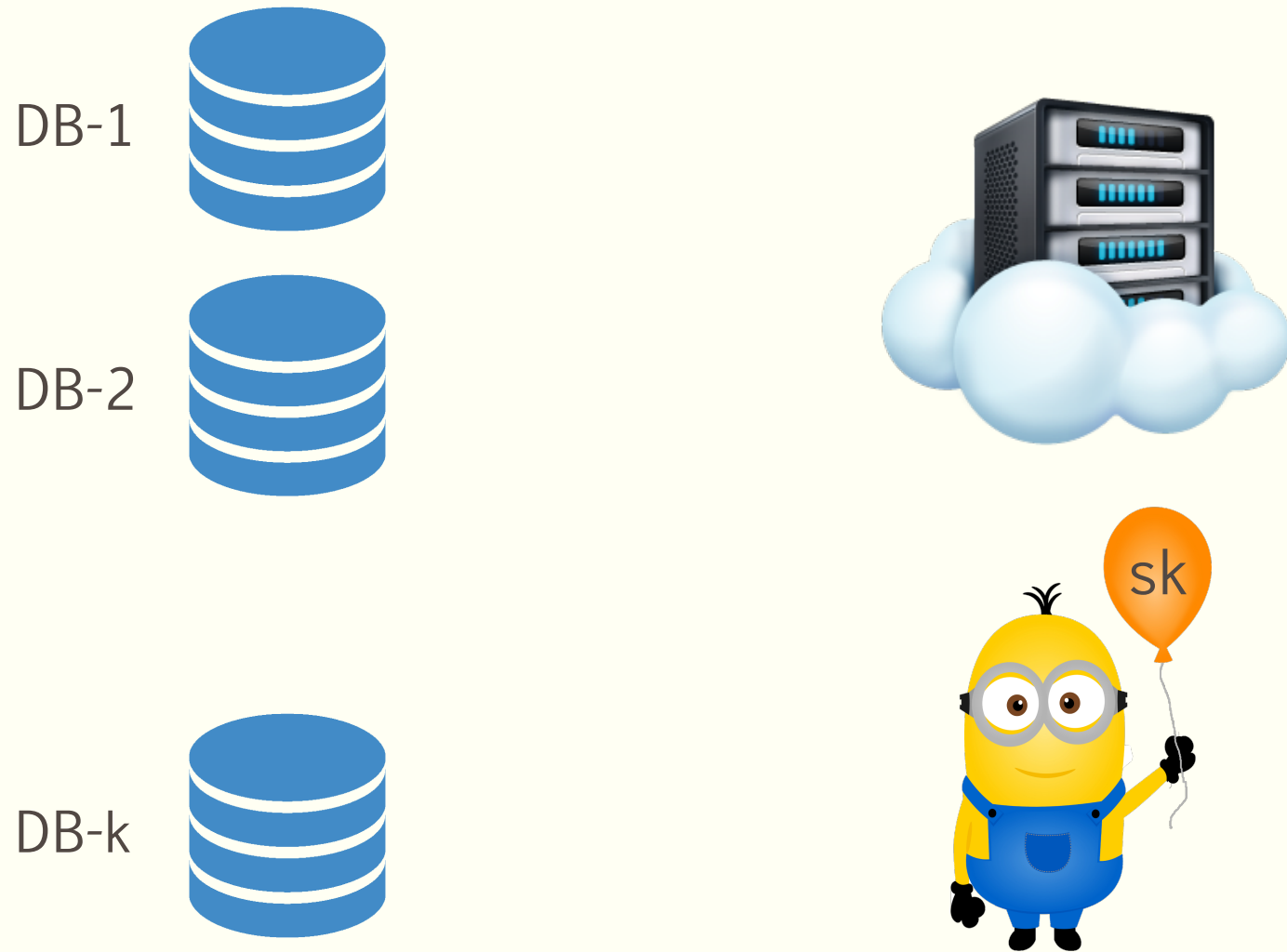
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- Protecting Computation, not just data
- Differential Privacy
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- ...
- Homomorphic Encryption (2009~)



# Homomorphic Encryption

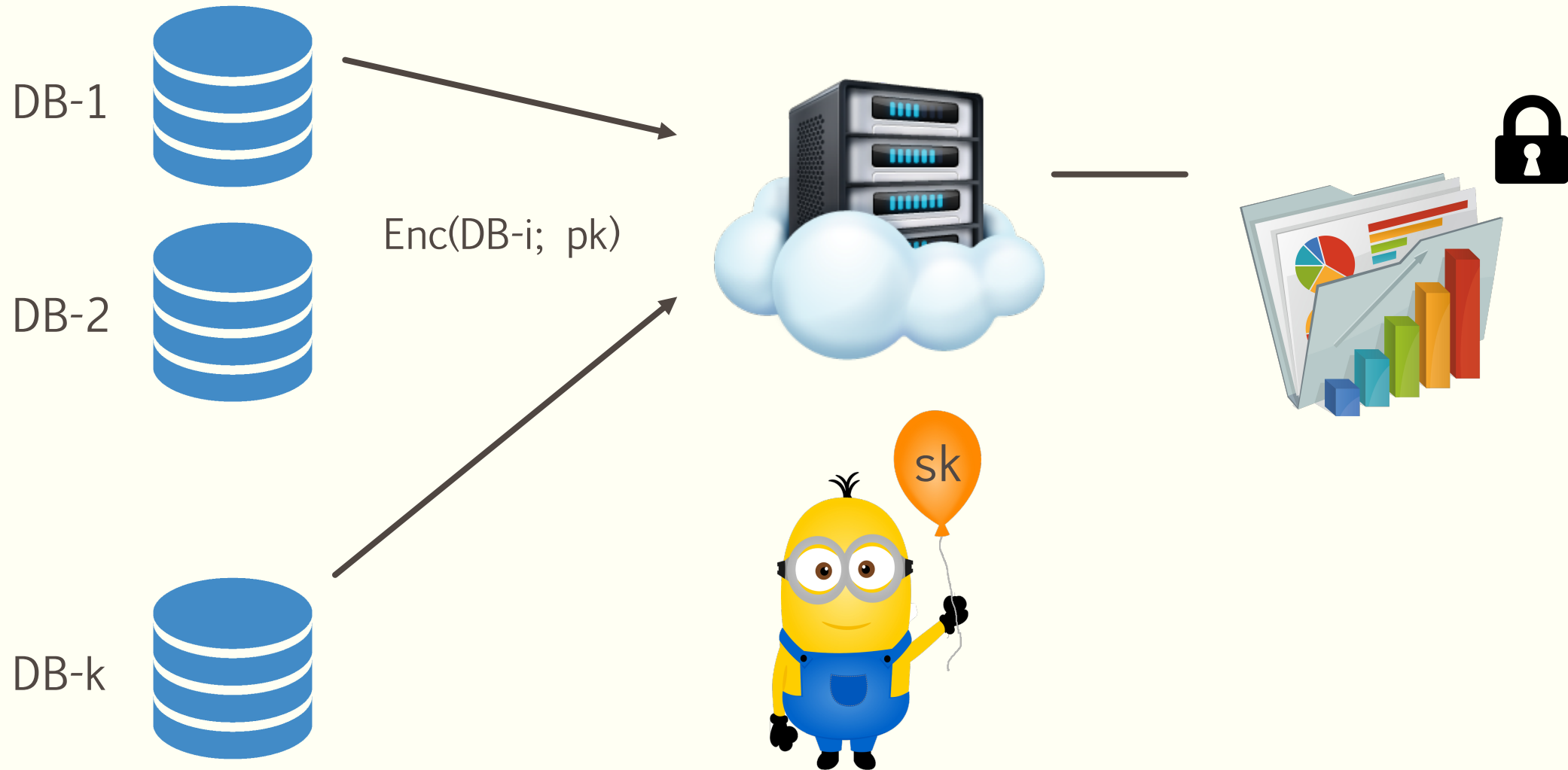
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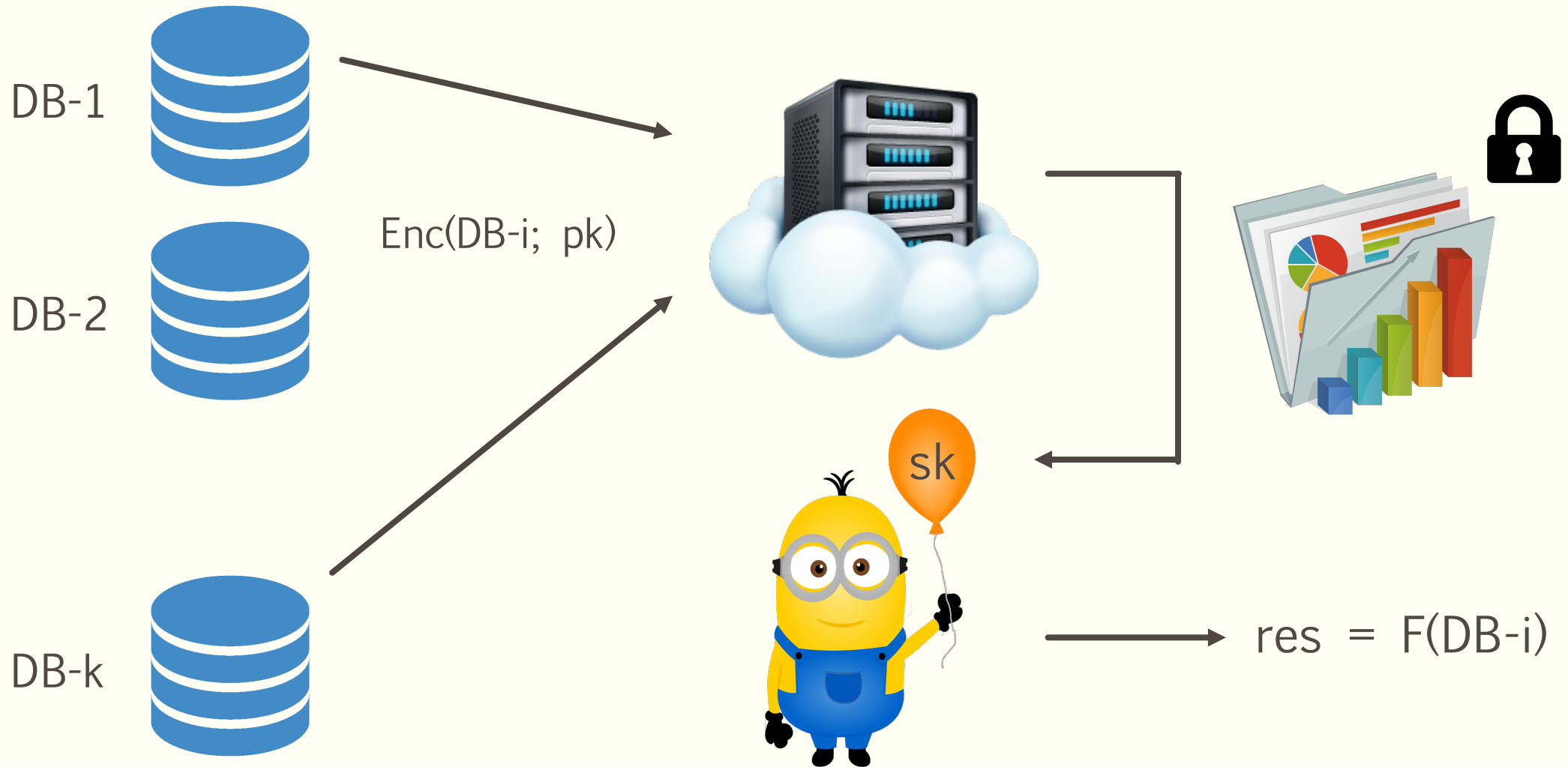
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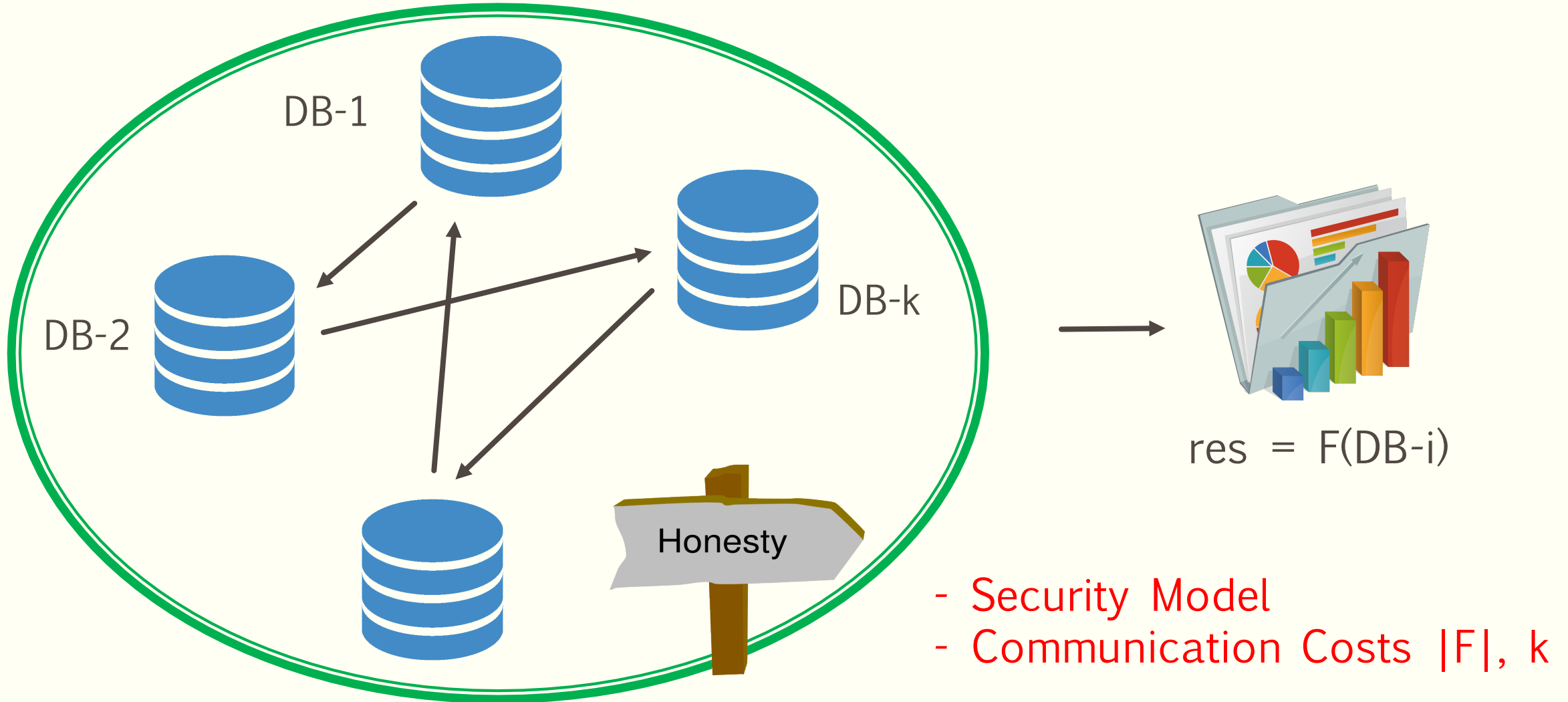
# Homomorphic Encryption

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# Multi-Party Computation

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# Comparison: HE vs MPC

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	Homomorphic Encryption	Multi-Party Computation
<b>Re-usability</b>	One-time encryption No further interaction	Single-use encryption Interaction between parties each time
<b>Model</b>	Semi-honest Cloud + Trusted SK Owner	Semi-honest parties without collusion
<b>Speed</b>	Slow in computation (but can speed-up using SIMD)	Slow in communication (due to large circuit to be exchanged)

# Summary of Progresses

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- 2009-10: Plausibility
  - [GH11] A single bit operation takes 30 minutes
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  - [GHS12b] 120 blocks of AES-128 (30K gates) in 36 hours



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  - [HS14] IBM's open-source library HElib
  - Implementation of Brakerski-Gentry-Vaikuntanathan (BGV) scheme
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- 2015-today: Usability
  - Various schemes with different advantages
  - Simpler and faster implementations
  - Real-world tasks: Big data analysis, Machine learning
  - Standardization meetings (2017~)
  - iDASH competitions (2014~)



# 4 Big Takeaways from Satya Nadella's Talk at Microsoft Build



By [JONATHAN VANIAN](#) May 7, 2018

[Microsoft](#) CEO Satya Nadella is trying to distinguish the business technology giant from its technology brethren by focusing on digital privacy.

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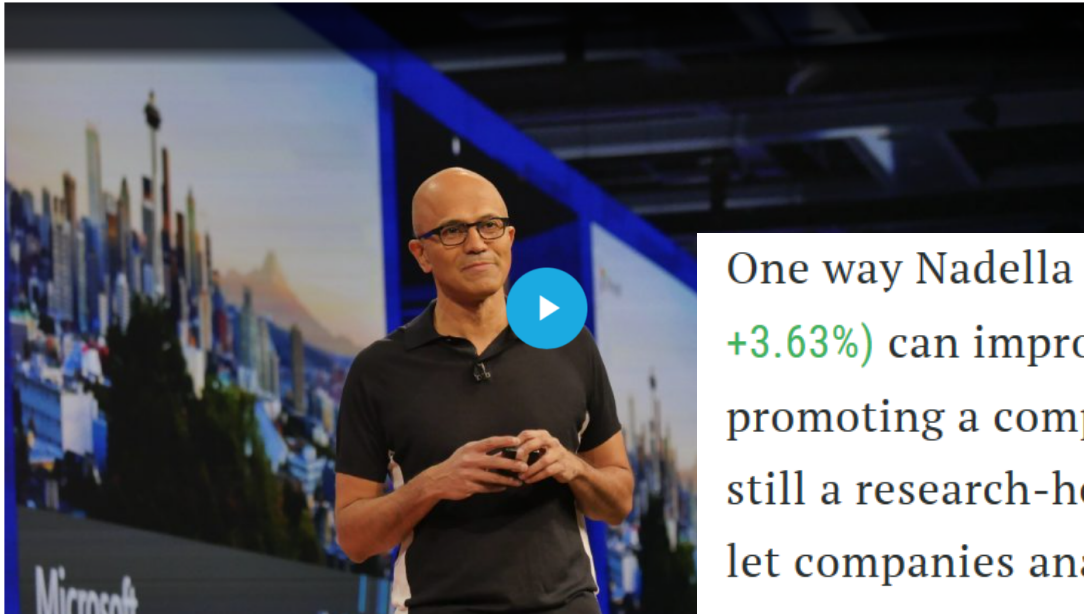
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[Microsoft](#) CEO Satya Nadella is trying to distinguish the giant from its technology brethren by focusing on digital

One way Nadella is attempting to convince businesses that Microsoft ([MSFT](#), [+3.63%](#)) can improve its AI technology while protecting user data is by promoting a computing technique called homomorphic encryption. Although still a research-heavy technique, [homomorphic encryption](#) would presumably let companies analyze and crunch encrypted data without needing to unscramble that information.

Nadella is pitching the technique as a way for companies to [“learn, train on encrypted data.”](#) The executive didn’t explain how far along Microsoft is on advancing the encryption technique, but the fact that he mentioned the wonky terms shows that the company is touting user privacy as a selling point for its Azure cloud business.

# Best Performing HE Schemes

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Type	Classical HE	Fast Bootstrapping	Approximate Encryption
Scheme	[BGV12] BGV [Bra12, FV12] B/FV	[DM15] FHEW [CGGI16] TFHE	[CKKS17] HEAAN
Plaintext			
Operation			
Library			

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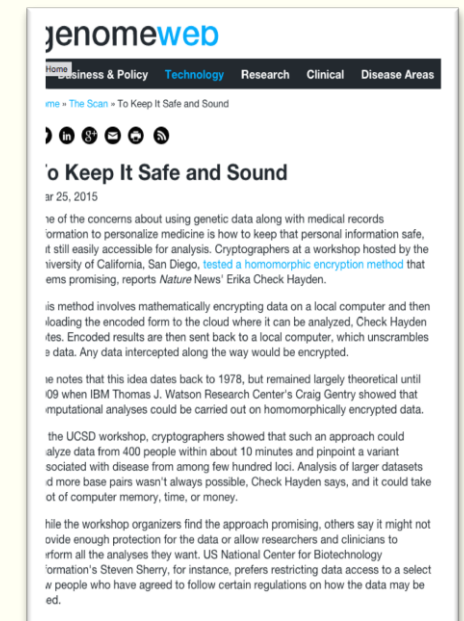
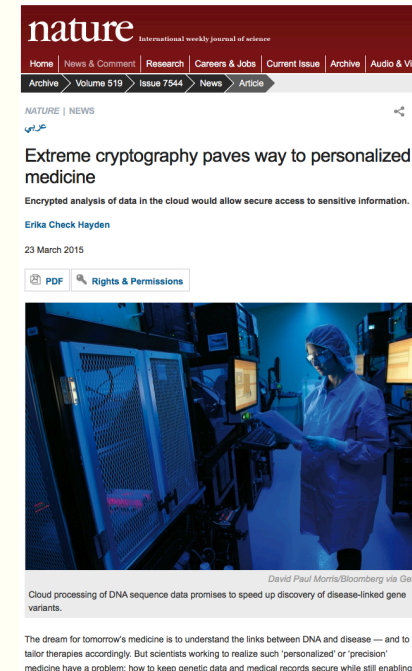
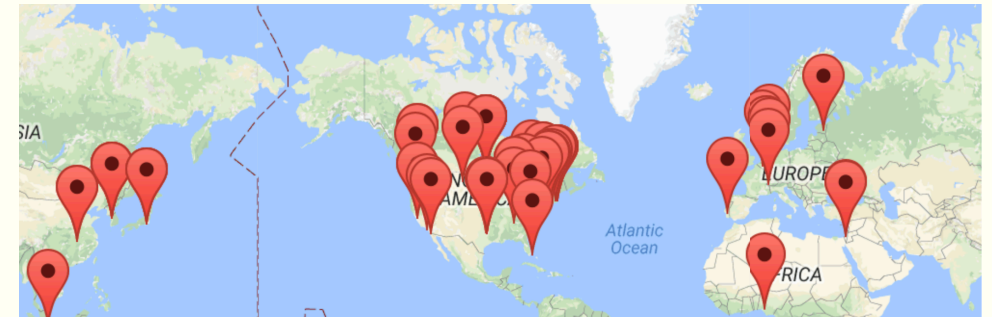
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Plaintext	Finite Field Packing	Binary string	Real/Complex numbers Packing
Operation	Addition, Multiplication	Look-up table & bootstrapping	Fixed-point Arithmetic
Library	HElib (IBM) SEAL (Microsoft Research) Palisade (Duality inc.)	TFHE (inpher, gemalto, etc.)	HEAAN (SNU)

# iDASH Security & Privacy Workshop

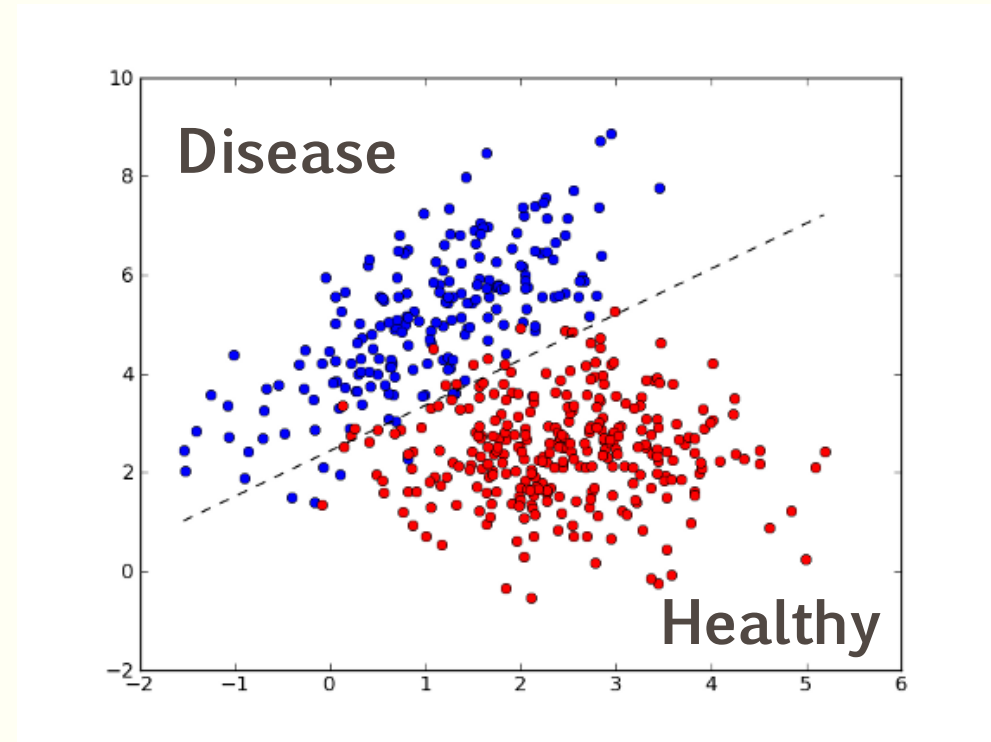
- An interdisciplinary challenge on genomic privacy research
- Motivated by real world biomedical applications
- Participation of privacy technology experts (academia and industry)
- Developed practical yet rigorous solutions for privacy preserving genomic data sharing and analysis
- Reported in the media (e.g., Nature News, GenomeWeb)



# iDASH 2017 – Logistic Regression Model Training

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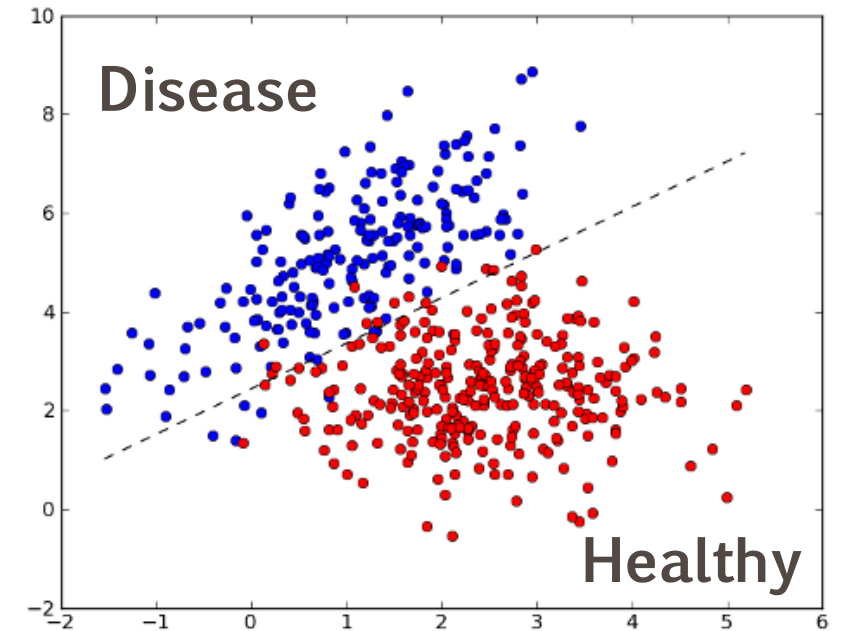
- A machine learning model to predict the disease
- 1500 records + 18 features for training



# iDASH 2017 – Logistic Regression Model Training

- A machine learning model to predict the disease
- 1500 records + 18 features for training

Teams	AUC 0.7136	Secure learning		Overall time (mins)
		Time (mins)	Memory (MB)	
SNU	<b>0.6934</b>	10.250	2775.333	<b>10.360</b>
CEA LIST	<b>0.6930</b>	2206.057	238.255	<b>2207.363</b>
KU Leuven	<b>0.6722</b>	155.695	7266.727	<b>160.912</b>
EPFL	<b>0.6584</b>	15.089	1498.513	<b>16.739</b>
MSR	<b>0.6574</b>	385.021	26299.344	<b>396.390</b>
Waseda*	<b>0.7154</b>	2.077	7635.600	<b>5.332</b>
Saarland**	N/A	48.356	29752.527	<b>57.344</b>



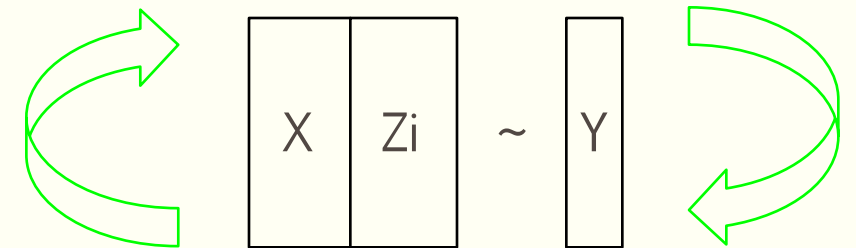
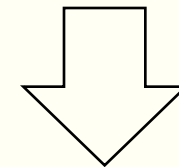
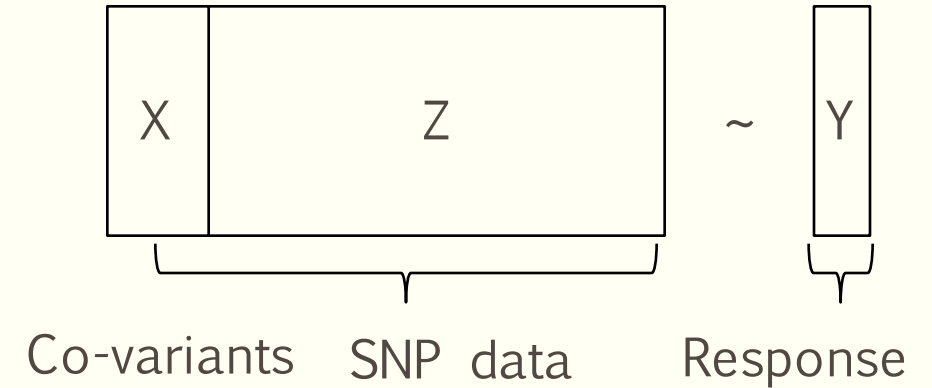
\* Interactive mechanism, no complete guarantee on 80-bit security at “analyst” side



# iDASH 2018 – Semi-Parallel GWAS

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- Compute Genome Wide Association Studies (GWAS)
- 3 Co-variants [age, height, weight] + 14,841 SNPS

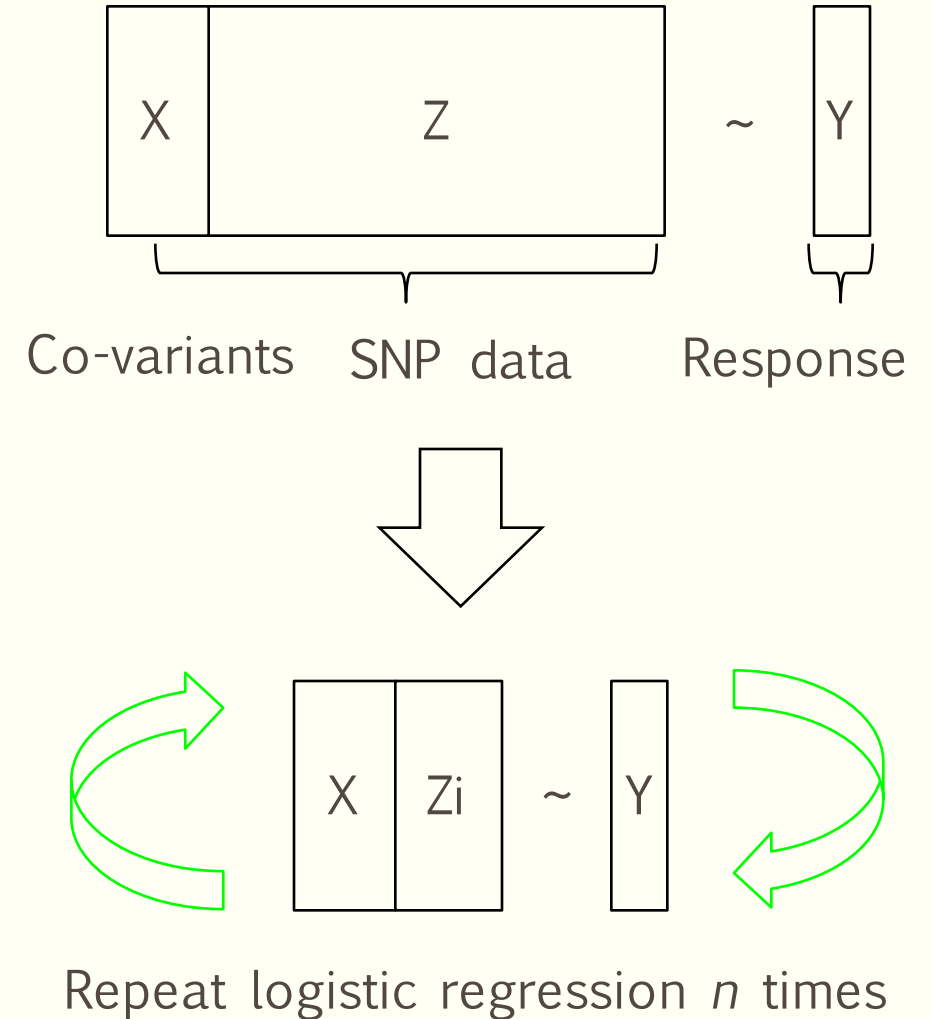


Repeat logistic regression  $n$  times

# iDASH 2018 – Semi-Parallel GWAS

- Compute Genome Wide Association Studies (GWAS)
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Team	Submission	Schemes	Time (mins)	Memory (MB)	Accuracy
A*FHE	A*FHE 1	HEAAN	922.48	3,777	0.999
	A*FHE 2		1,632.97	4,093	0.905
Chimera	Version 1	TFHE+HEAAN	201.73	10,375	0.993
	Version 2	(Chimera)	215.95	15,166	0.35
Delft Blue	Delft Blue	HEAAN	1,844.82	10,814	0.969
UCSD	Log Reg	HEAAN	1.66	14,901	0.993
	Lin Reg	pkg: RNS HEAAN	0.42	3,387	0.989
Duality Inc	Log Reg	HEAAN	3.80	10,230	0.993
	Chi2 test	pkg: PALISADE	0.09	1,512	0.983
SNU	SNU 1	HEAAN	52.49	15,204	0.984
	SNU 2		52.37	15,177	0.988
IBM	IBM-Complex	HEAAN	23.35	8,651	0.911
	IBM- Real	pkg: HEllb	52.65	15,613	0.526



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# Approximate Computation

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- Numerical Representation

Encode  $m$  into an integer  $m \approx px$  for a scaling factor  $p$

$$\sqrt{2} \mapsto 1412 \approx \sqrt{2} \cdot 10^3$$

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- Fixed-Point Multiplication

Compute  $m = m_1 m_2$  and extract its significant digits  $m' \approx p^{-1} \cdot m$

$$1.234 \times 5.678 = (1234 \cdot 10^{-3}) \times (5678 \cdot 10^{-3}) = 7006652 \cdot 10^{-6} \mapsto 7007 \cdot 10^{-3} = 7.007$$

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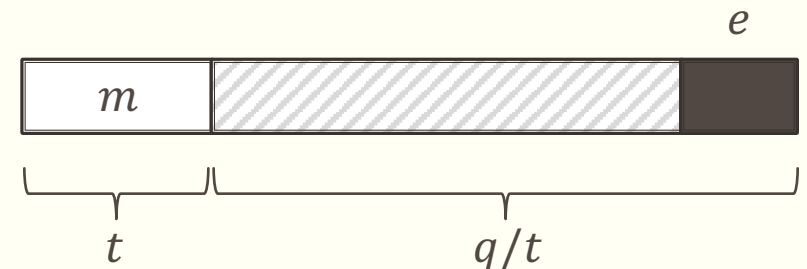
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- Previous (LWE-based) HE

$$\text{ct} = \text{Enc}_{\text{sk}}(m), \quad \langle \text{ct}, \text{sk} \rangle = \frac{q}{t} m + e \pmod{q}$$

Modulo  $t$  plaintext vs Rounding operation



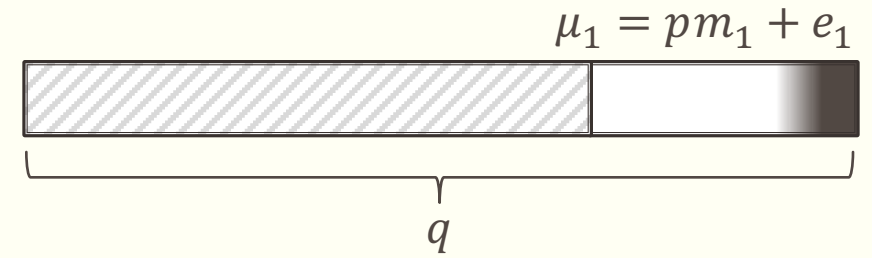
# HEAAN

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- A New Message Encoding

$$ct = \text{Enc}_{sk}(m), \quad \langle ct, sk \rangle = pm + e \pmod{q}$$

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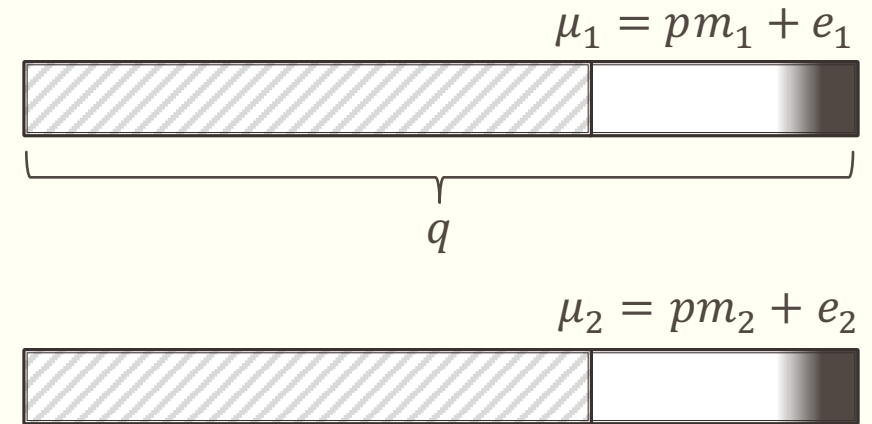
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- Homomorphic Operations

Input:  $\mu_1 \approx pm_1, \mu_2 \approx pm_2$

Addition:  $\mu_1 + \mu_2 \approx p \cdot (m_1 + m_2)$





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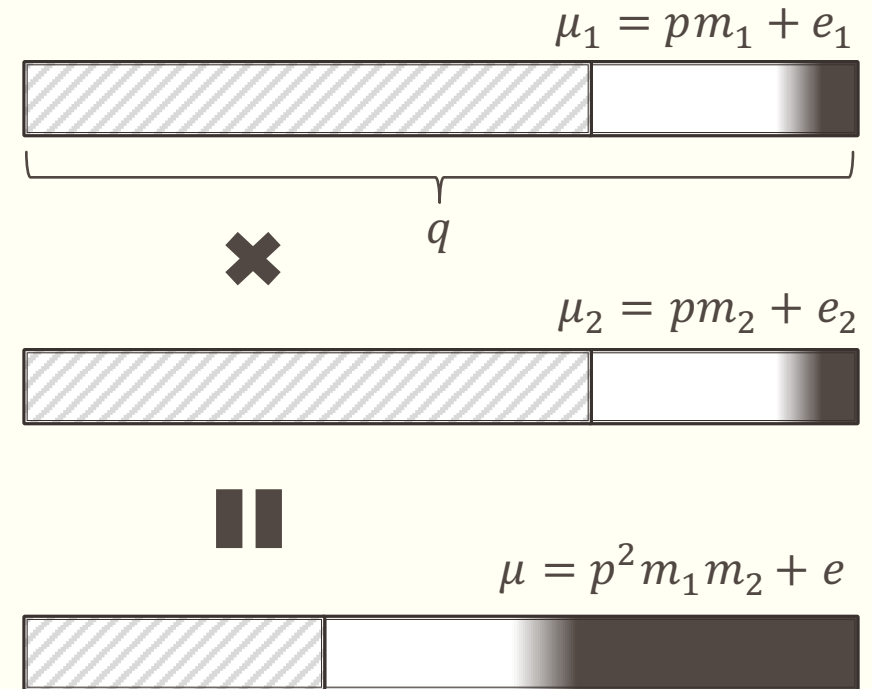
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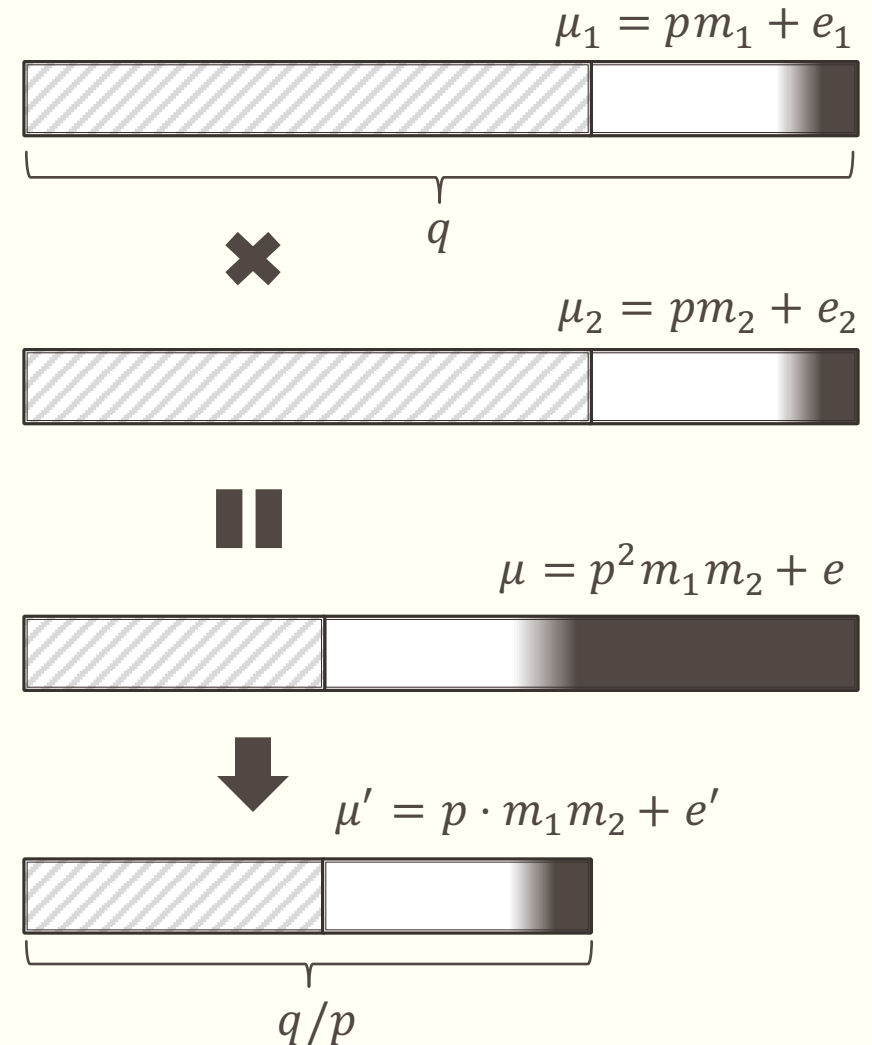
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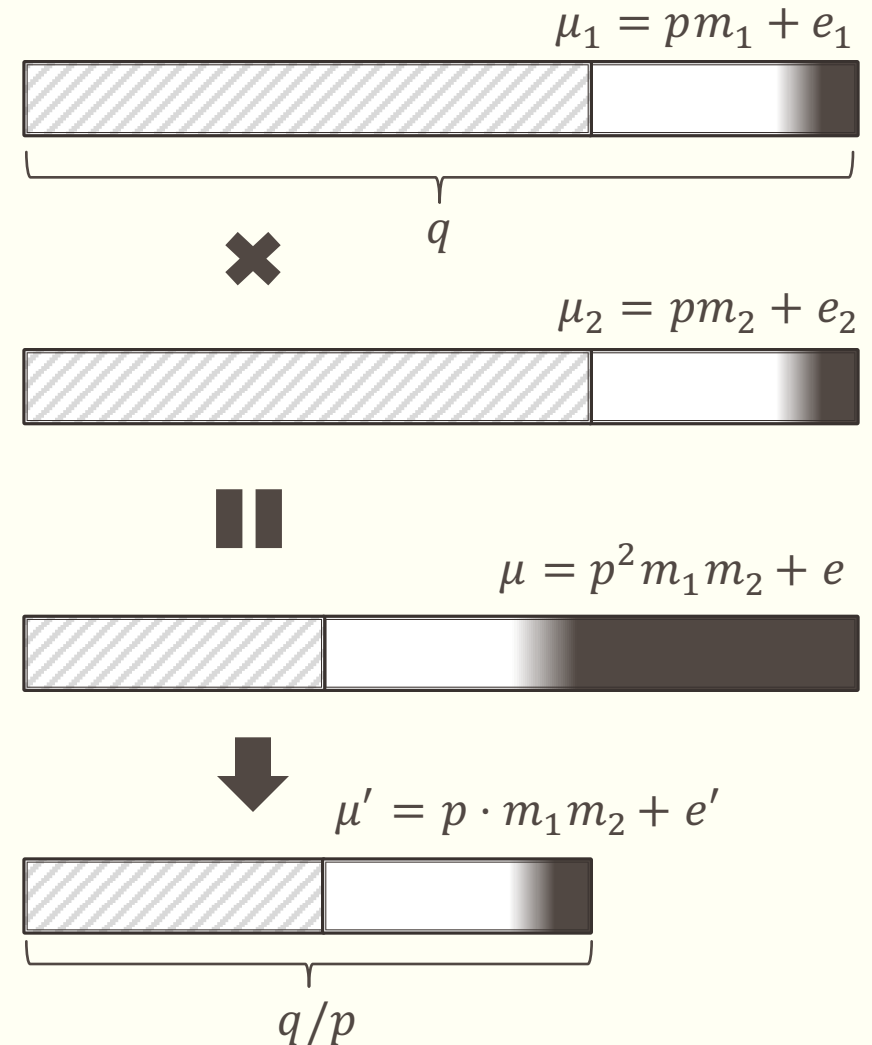
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- Support for the (approximate) fixed-point arithmetic !

- **Leveled** HE :  $q = p^L$



# Packed Ciphertext

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  - Parallel computation in a SIMD manner  $z \otimes w = (z_1 w_1, z_2 w_2, \dots, z_\ell w_\ell)$

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- RLWE-based HEAAN
  - A ciphertext can encrypt a polynomial  $m(X) \in R$
  - Observation:  $X^n + 1 = (X - \zeta_1)(X - \zeta_1^{-1})(X - \zeta_2)(X - \zeta_2^{-1}) \dots (X - \zeta_{n/2})(X - \zeta_{n/2}^{-1})$

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- Permutation of plaintext slots

$$\text{Rotate: } \text{Enc}(z_1, z_2, \dots, z_{n/2}) \mapsto \text{Enc}(z_2, \dots, z_{n/2}, z_1)$$



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- Bootstrapping
  - Ciphertexts of a leveled HE have a limited lifespan
  - Refresh a ciphertext  $ct = \text{Enc}_{sk}(m)$  by **evaluating the decryption circuit homomorphically**

$$\text{Dec}_{sk}(ct) = m \iff F(sk) = m \text{ where } F(*) = \text{Dec}_*(ct)$$

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- HEAAN

- Homomorphic operations introduce errors

$$F(BK) = F(\text{Enc}_{sk}(sk)) = \text{Enc}_{sk}(F(sk) + e) = \text{Enc}_{sk}(m + e)$$

- It is ok to have an additional error
- How to evaluate the decryption circuit efficiently?

$$\text{Dec}_{sk}(ct) = \langle ct, sk \rangle \pmod{q}$$

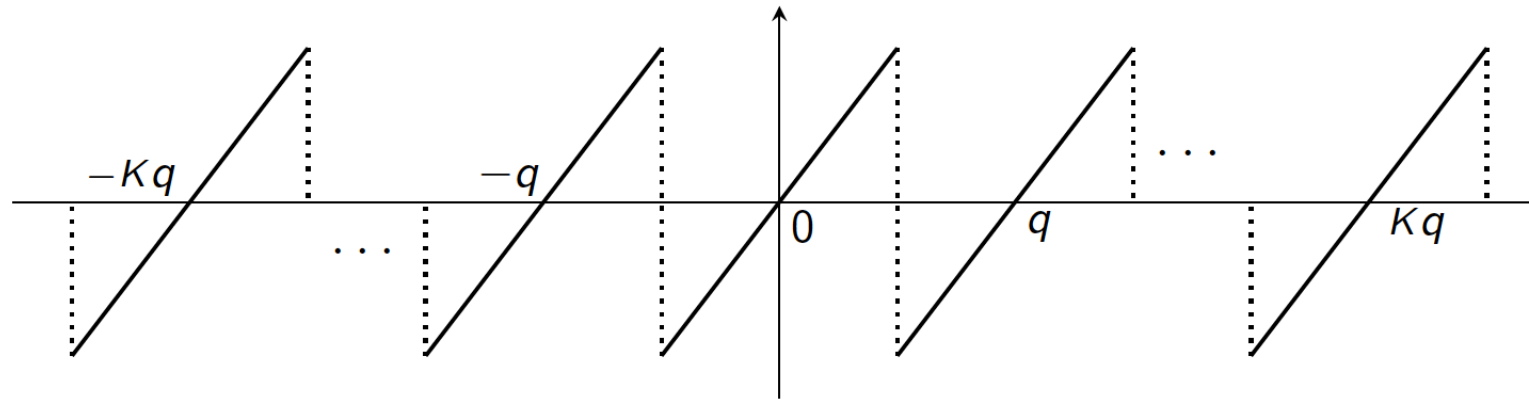
# Approximate Decryption

---

$$\text{Dec}_{\text{sk}}(\text{ct}) \mapsto t = \langle \text{ct}, \text{sk} \rangle \mapsto [t]_q = \mu,$$

$$t = qI + \mu \text{ for some } |I| < K$$

- Naive solution: polynomial interpolation on  $[-Kq, Kq]$
- Huge depth, complexity & inaccurate result



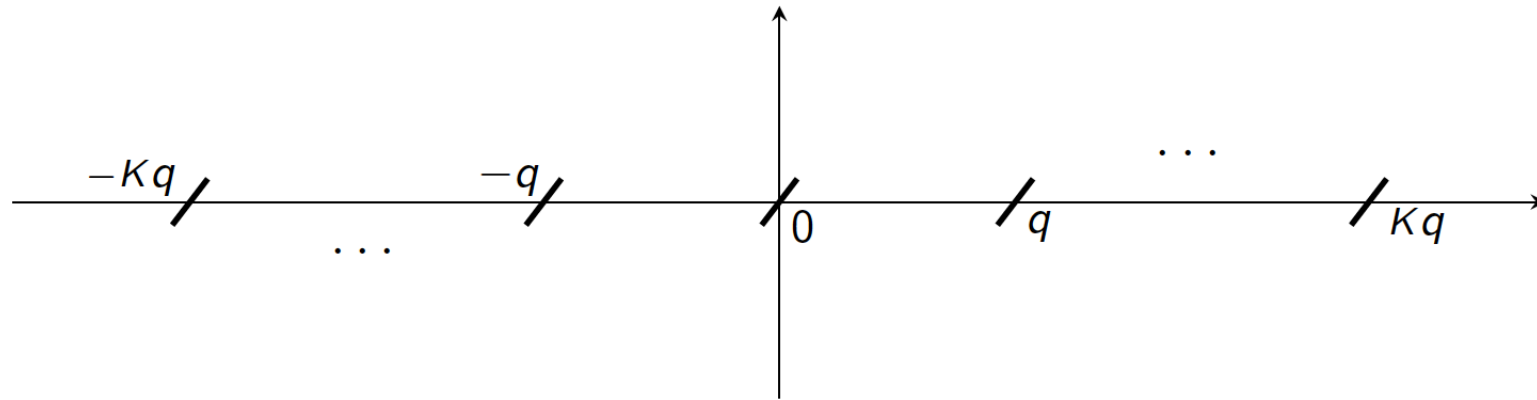
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- Idea 1: Restriction of domain  $|\mu| \ll q$



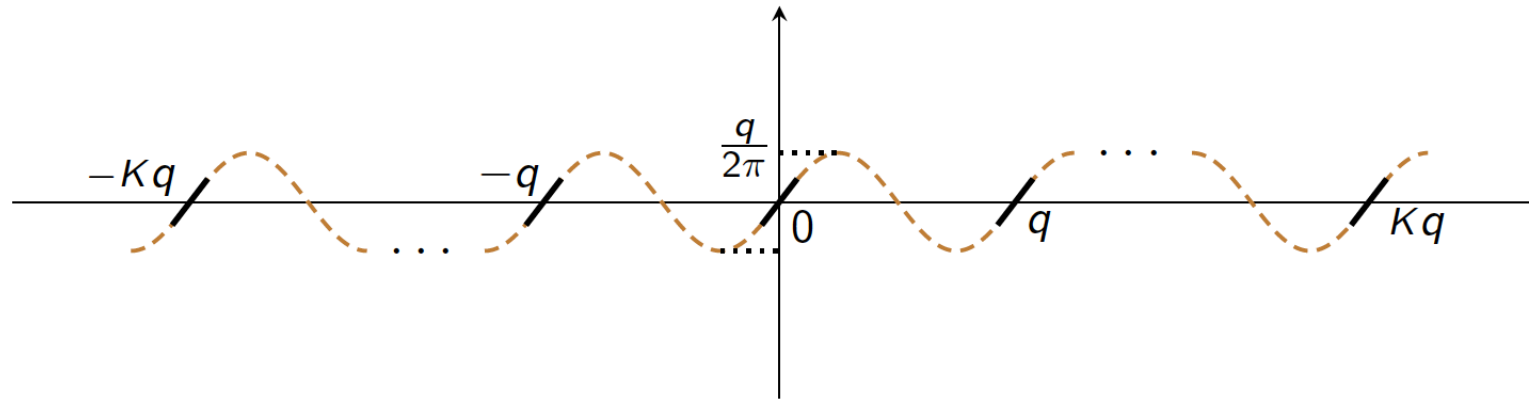
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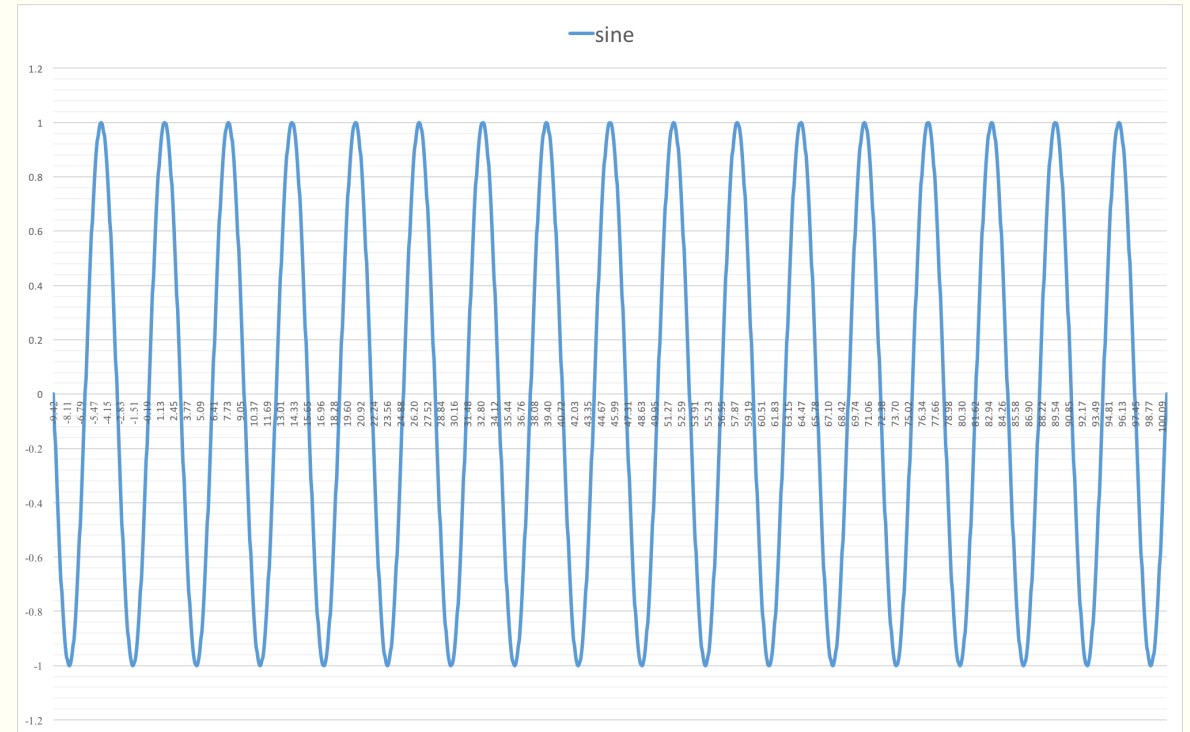
$$t = qI + \mu \text{ for some } |I| < K$$

- Idea 1: Restriction of domain  $|\mu| \ll q$
- Idea 2: Sine approximation  $\mu \approx \frac{q}{2\pi} \sin \theta$  for  $\theta = \frac{2\pi}{q} t$



# Sine Evaluation

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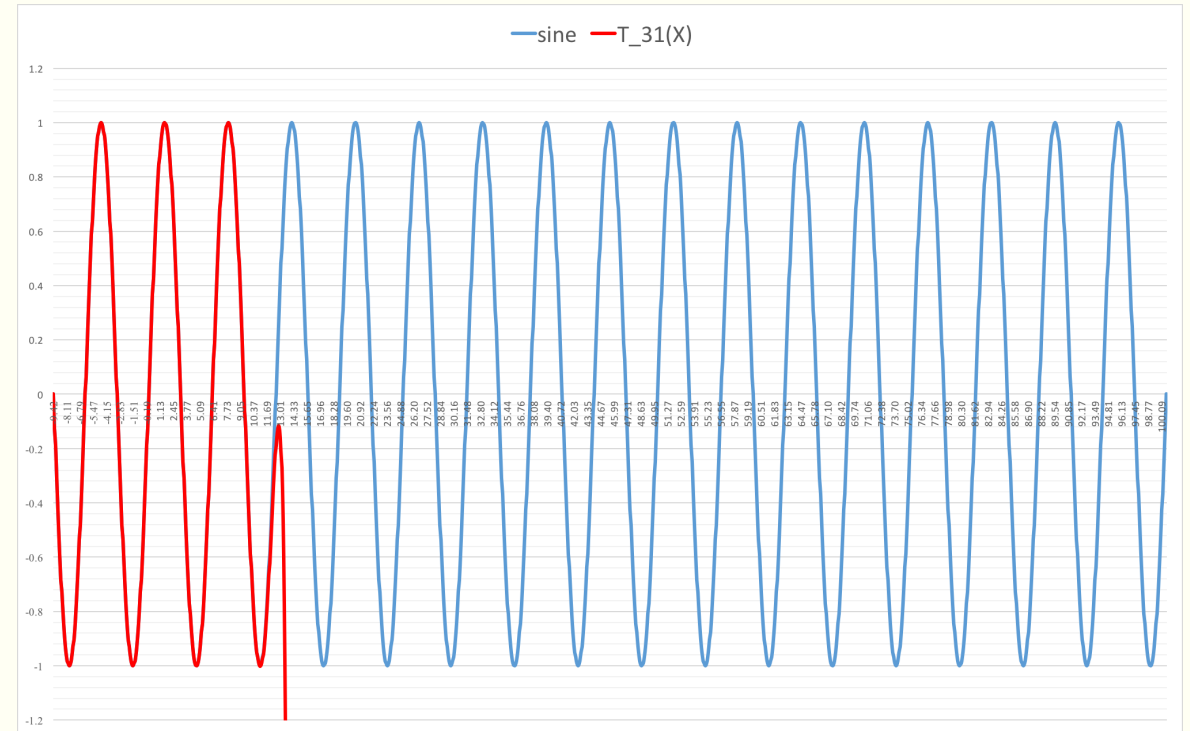




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  - huge depth & complexity, low precision



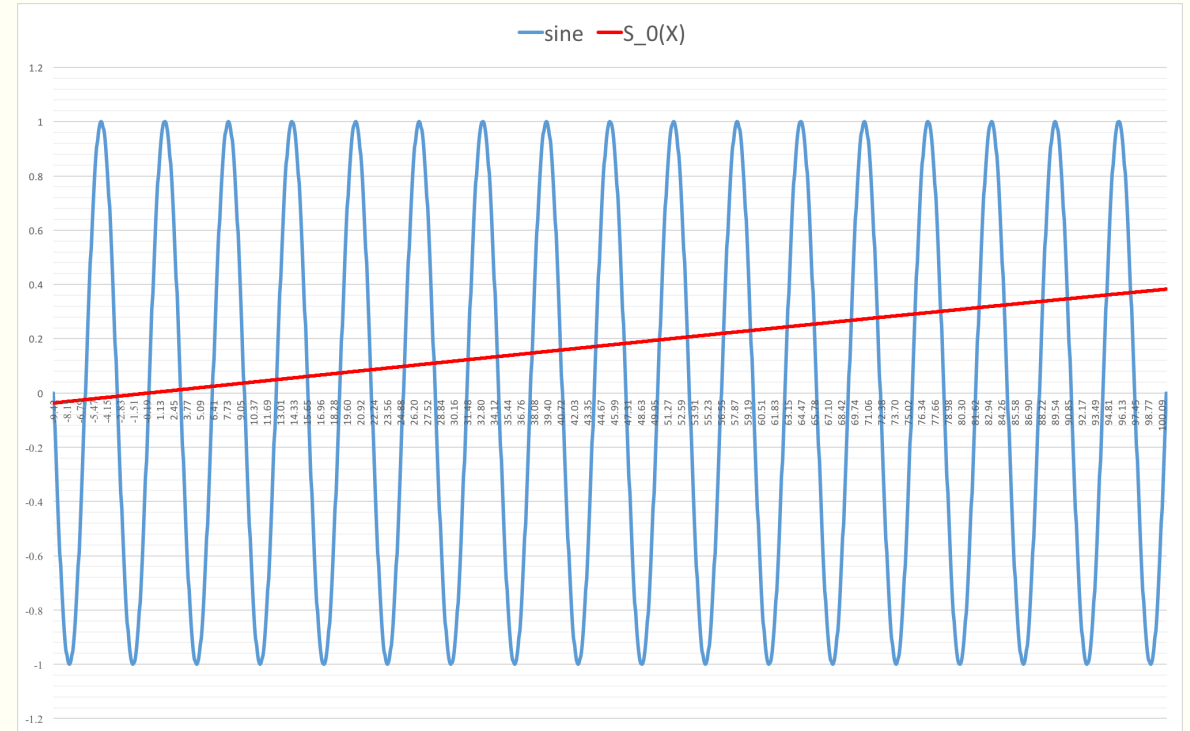
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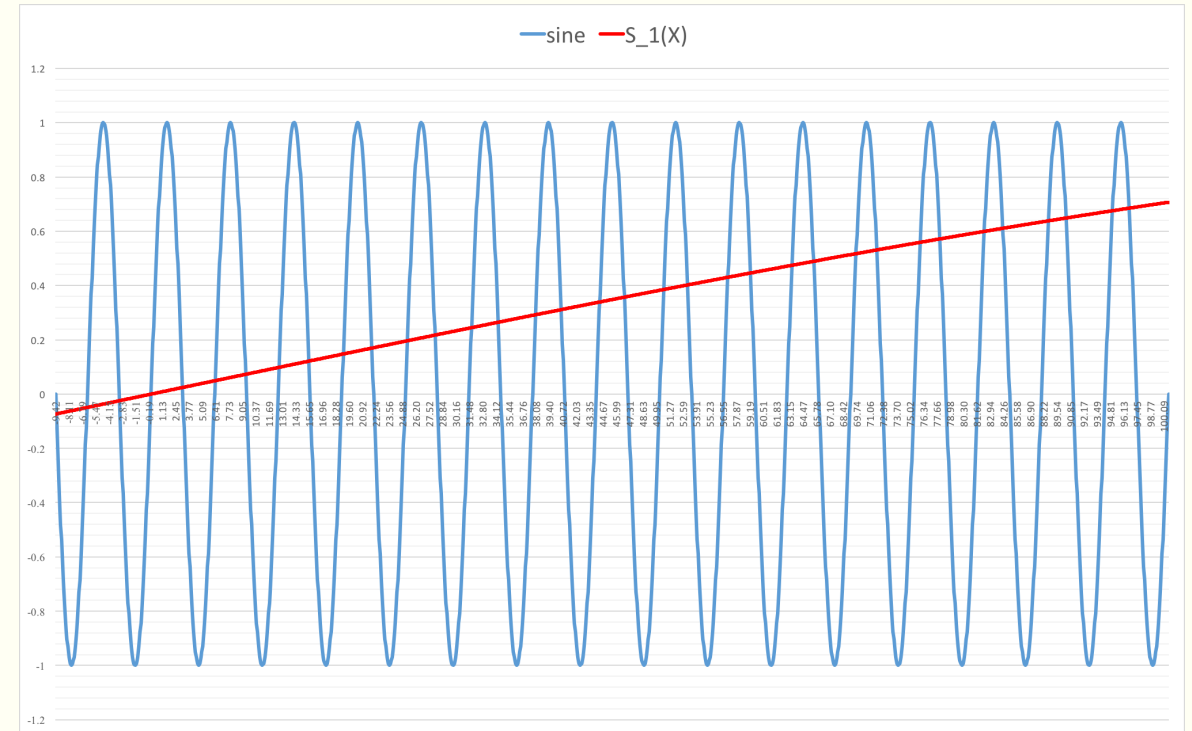
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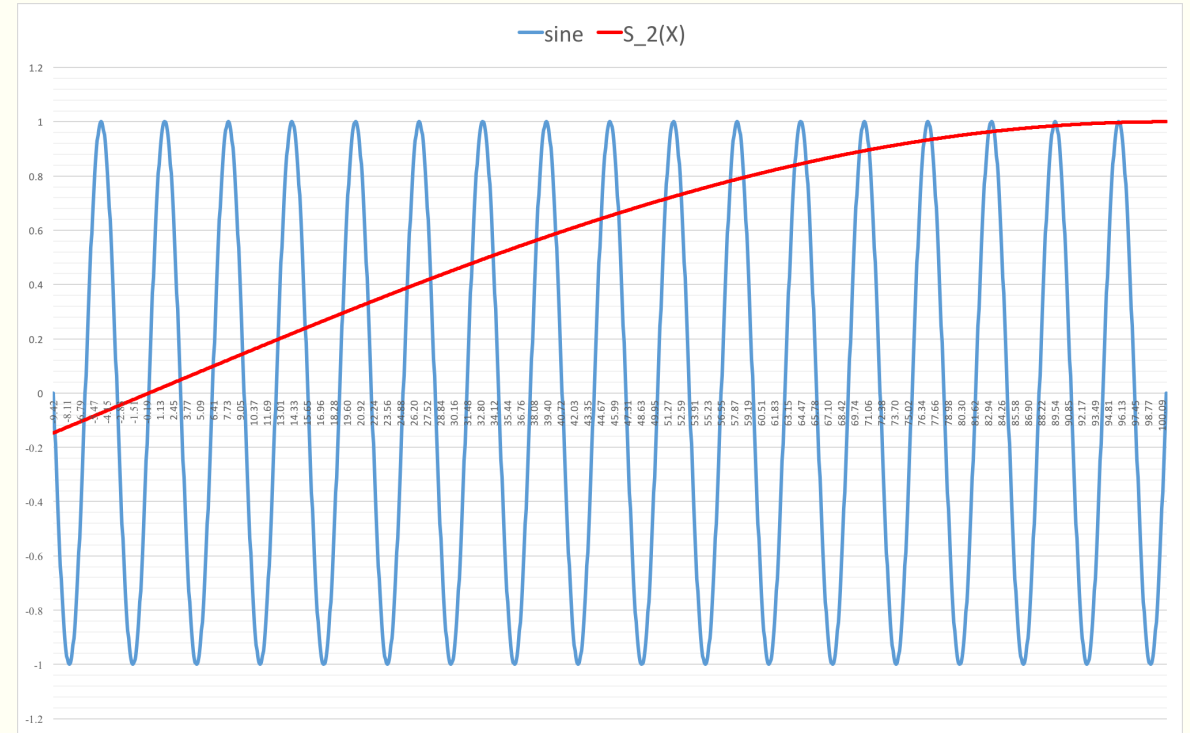
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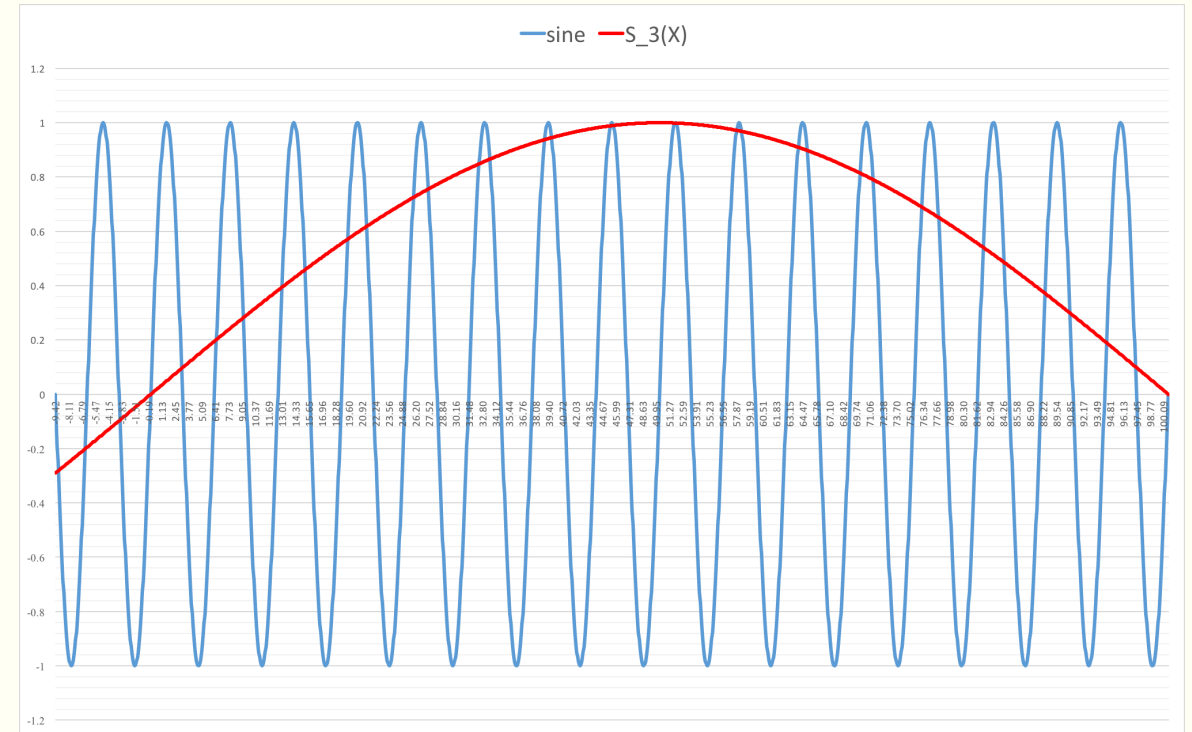
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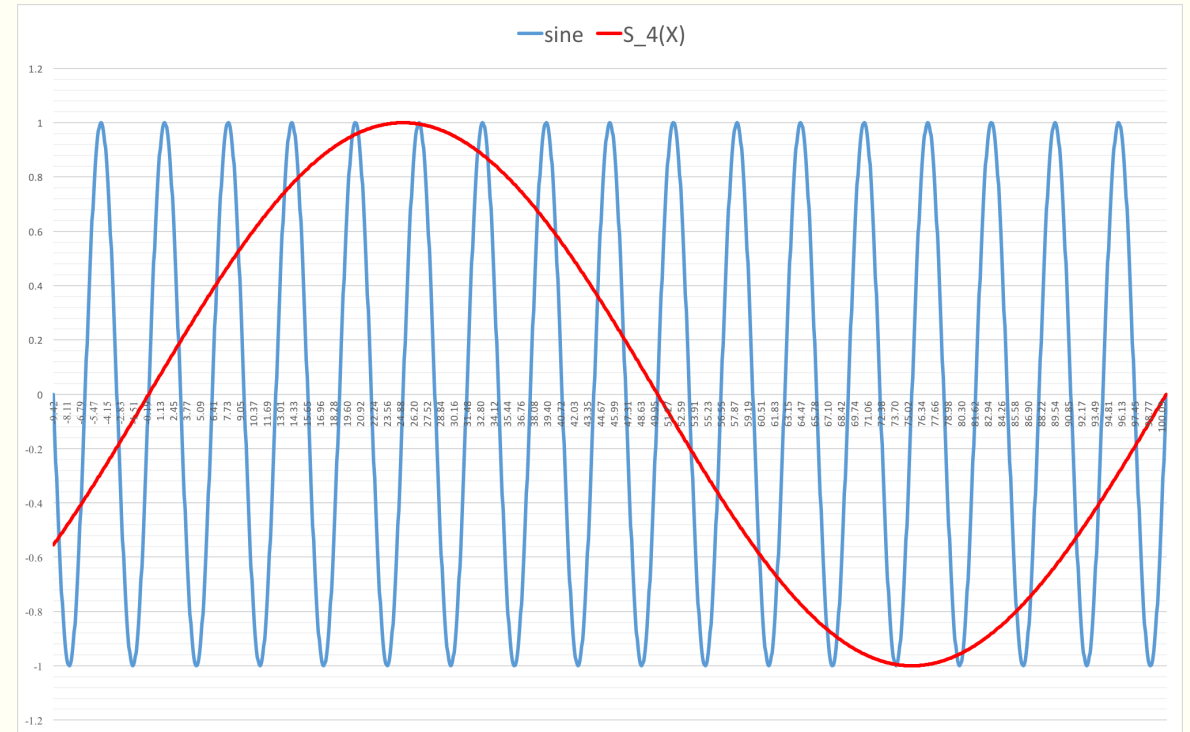
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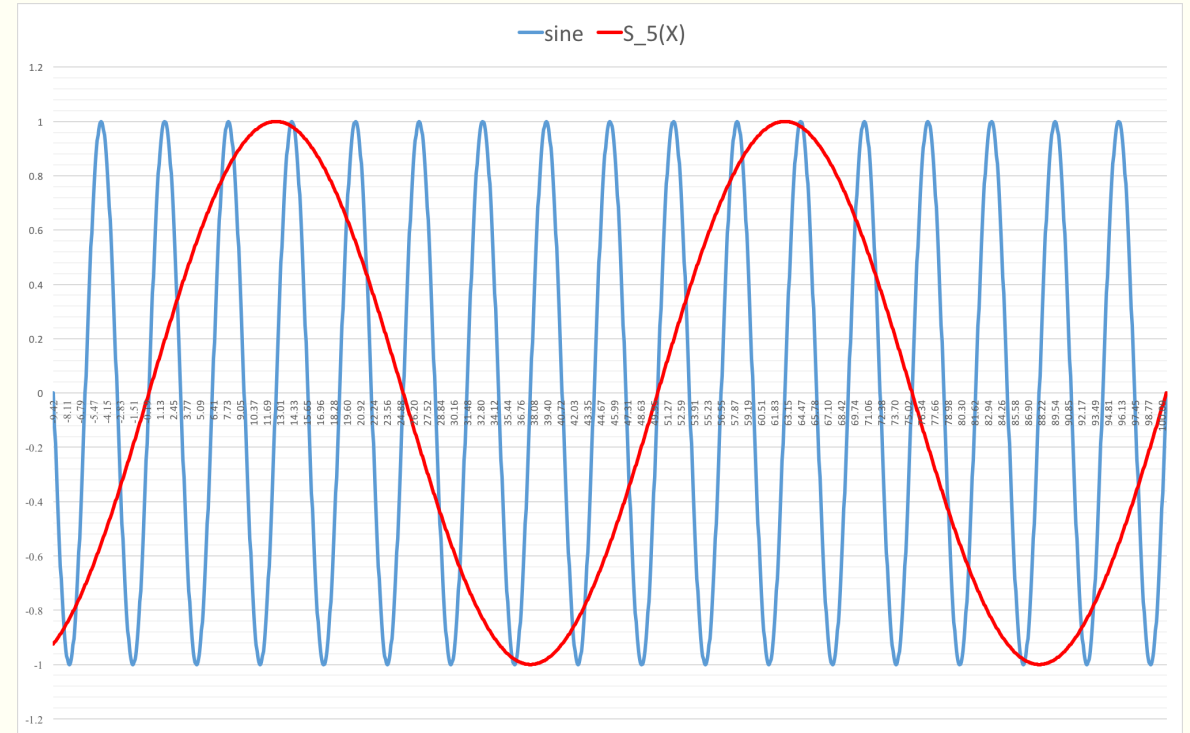
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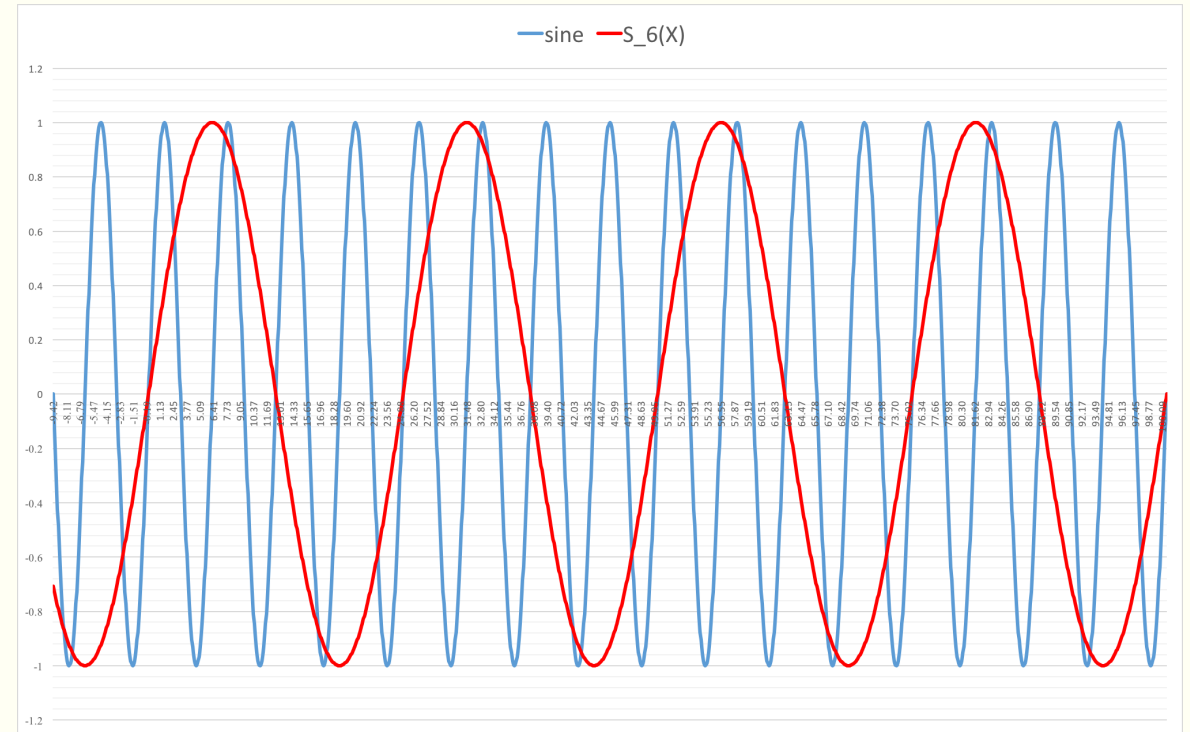
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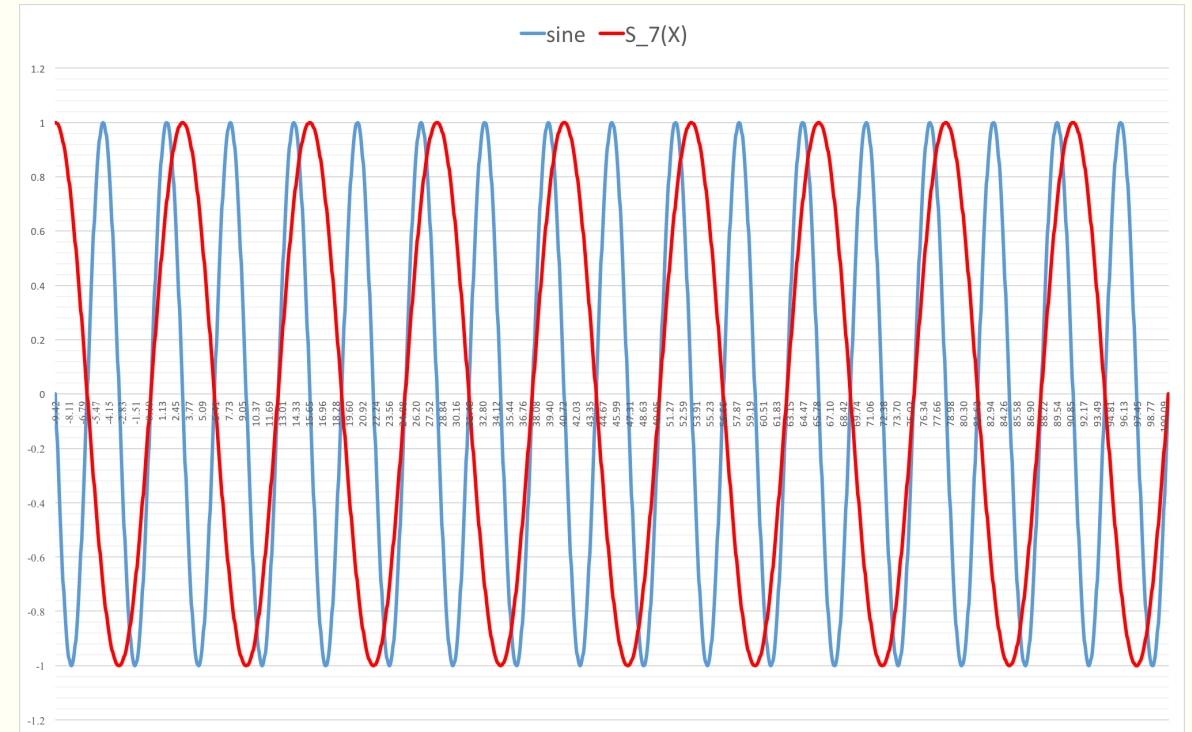
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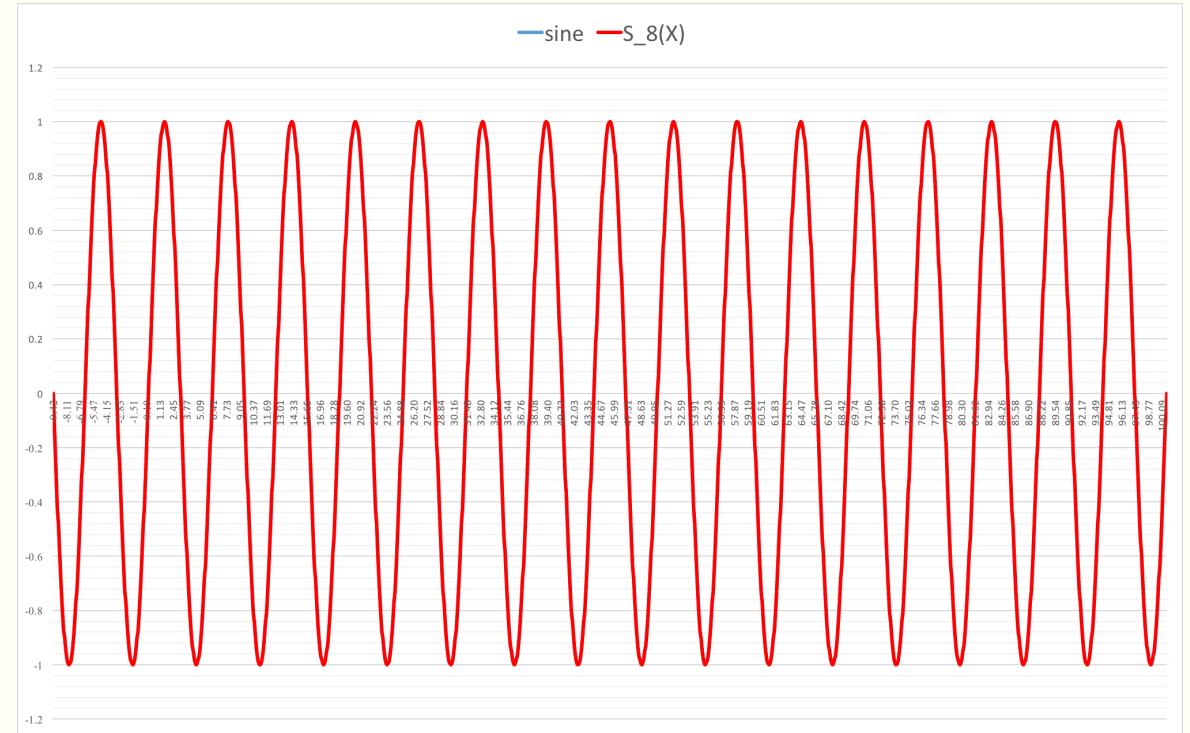
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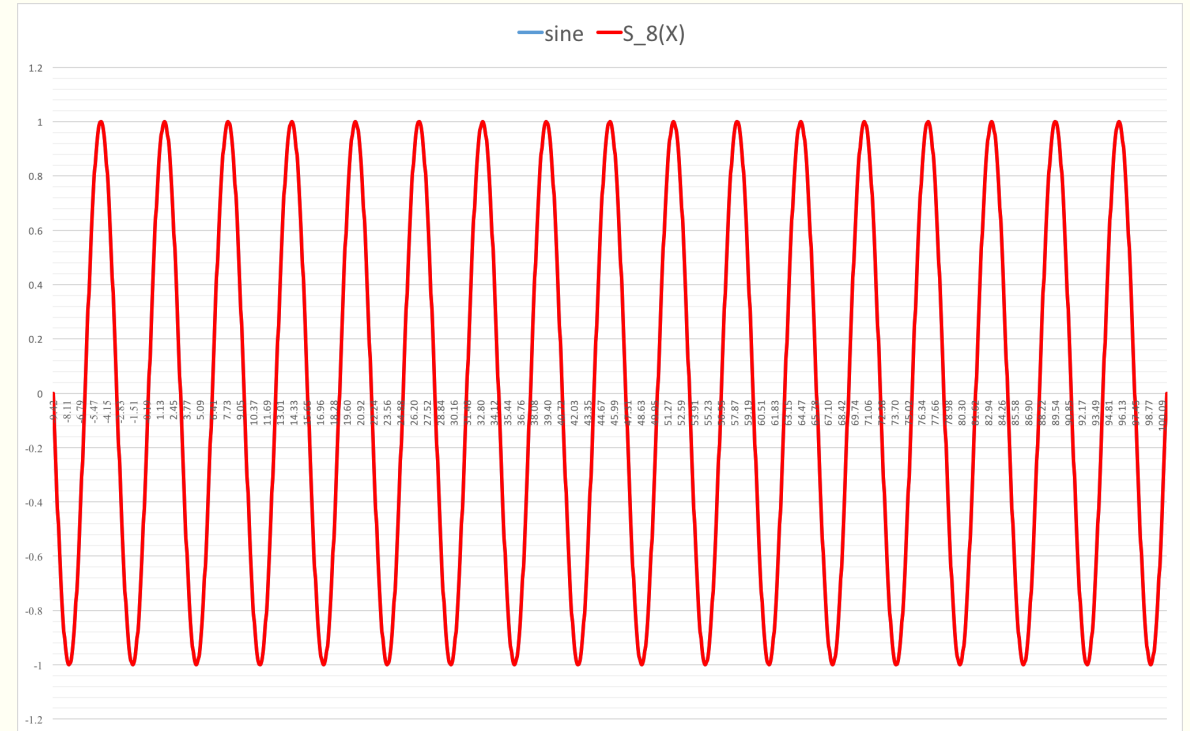
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- Numerically stable & Linear complexity



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# Slot-Coefficient Switching

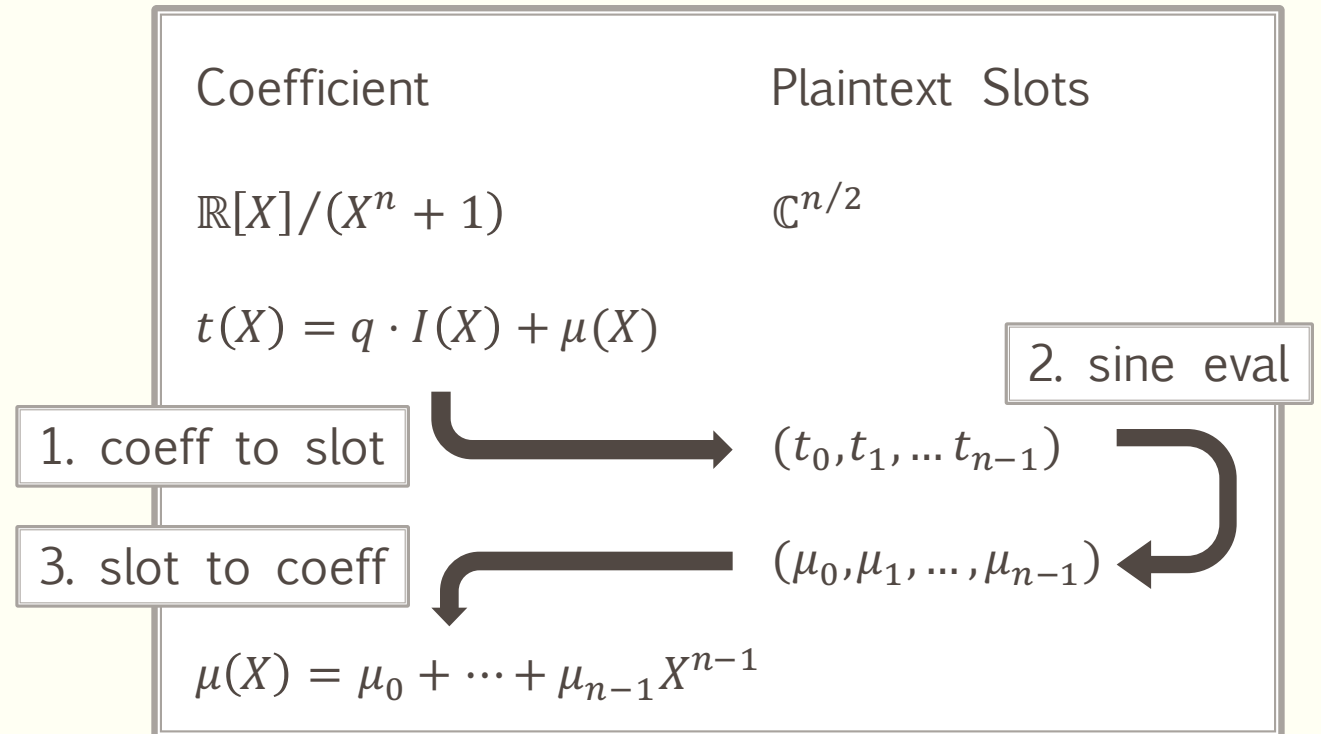
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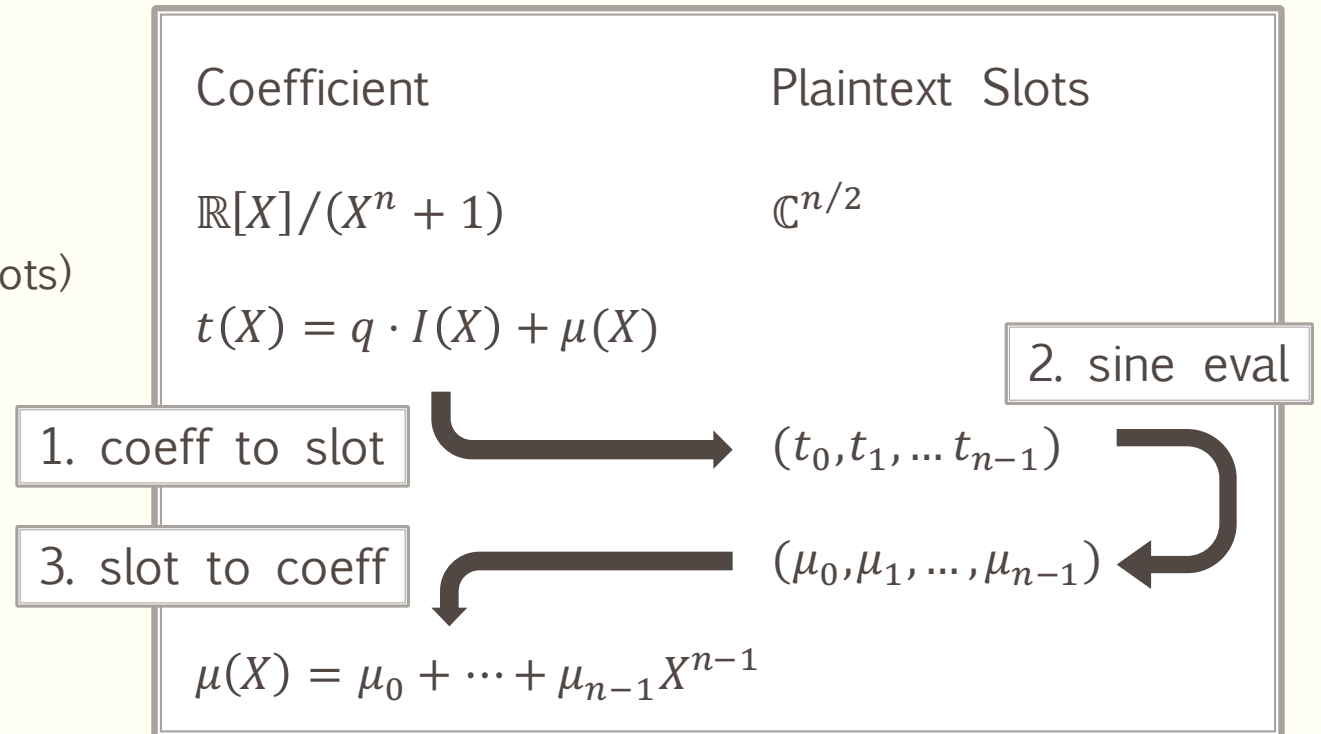
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- Performance of Bootstrapping
  - Depth consumption : Sine evaluation
  - Complexity: Slot-Coefficient switchings (# of slots)

- Experimental Results
  - $127 + 12 = 139$  s / 128 slots X 12 bits
  - $456 + 68 = 524$  s / 128 slots X 24 bits



# Table of Contents

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- Background
- Construction
  - [CKKS, AC17] Homomorphic Encryption for Arithmetic of Approximate Numbers
- Bootstrapping
  - [CHKKS, EC18] Bootstrapping for Approximate Homomorphic Encryption
- Related Works

# Followed Work

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- Improved Bootstrapping for Approximate Homomorphic Encryption
  - Joint work with Hao Chen and Ilaria Chillotti (submission to EC19)
  - FFT-like algorithms to optimize Slot-Coefficient switchings
  - Better evaluation of sine function based on Chebyshev approximation
- [JKLS, CCS18] Secure Outsourced Matrix Computation and Application to Neural Networks
  - Evaluation of an encrypted CNN model on the encrypted MNIST data
- [CHKKS, SAC18] A Full RNS Variant of Approximate Homomorphic Encryption
  - Better performance without any high-precision arithmetic library
  - iDASH 2018
- [KS, ICISC18] Approximate Homomorphic Encryption over the Real Numbers



